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Setting expansion of gypsum-bonded investment in dental casting

Part 2 *Setting expansion within an elastic ring*

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The setting expansion of gypsum-bonded investment has been measured within an elastic ring, and the equation to calculate the setting expansion has been obtained considering the effect of the mechanical properties of the ring and the ring radii on the setting expansion values. The result conclusively demonstrates that the calculated setting expansion values agree with the level measured for gypsum-bonded investment. The expansions calculated according to the equation below and those measured in an acrylic ring agree fairly well

$$U_i(r, t) = \frac{rLE'}{1 - \nu'}\left[1 - \exp\left\{-\frac{1-\nu'}{E'}\frac{a_0}{L}\left[1 - \exp(-kt)\right]\right\}\right]$$

$$L = \frac{(1-\nu)A^2 + (1+\nu)B^2}{E_r(B^2 - A^2)}$$

where $E' = 5\,\mathrm{kg\,cm^{-2}}$, $\nu' = 0.2$, $a_0 = 0.009$, and $k = 0.032\,\mathrm{min^{-1}}$, A and B denote the inner and outer radii, and ν and E_r Poisson's ratio and Young's modulus of the ring, respectively.

1. Introduction

The setting expansion of gypsum-bonded investment used for a dental casting is restricted as the investment slurry sets in a casting ring after mixing [1]. As previously reported, the change of setting expansion of the investment under uniaxial load with time has been calculated by a method developed by us [2], and the calculated value of setting expansion agreed fairly well with that measured with a dial gauge. In this study, the setting expansion within an acrylic ring was measured, and these results were compared with the values obtained using our calculation method.

2. Materials and methods

2.1. Tensile test

The value of E (Young's modulus) of the acrylic resin was measured using an Autograph DCS-5000 (Shimazu Co., Kyoto, Japan), and the effects of cross-head speed and thickness of the specimen on the value were investigated as shown in Fig. 1. The specimen had the dimensions 57.2 mm × 12.7 mm × 1.5 or 3.0 mm as the gauge length. The tensile cross-head speeds were 0.017 and $1.76\,\mathrm{mm\,min^{-1}}$. $0.017\,\mathrm{mm\,min^{-1}}$ corresponds to the deformation speed caused by setting expansion, and $1.76\,\mathrm{mm\,min^{-1}}$ corresponds to the speed for normal industrial tensile tests. The value of Poisson's ratio was determined to be 0.33 for acrylic resin [3]. The test was carried out at $17 \pm 2^\circ$C.

2.2. Measurement of setting expansion

The gypsum-bonded investment (a commercial one supplied by G-C Co., Tokyo, Japan) was used to obtain setting expansion within an acrylic ring. The acrylic rings had a constant height of 10 mm and an inner diameter of 44.0 mm, but they had variable outer diameters of 44.6, 46.0, and 50.0 mm in order to examine the effect of acrylic ring thickness on setting expansion.

The investment powder was mixed at a water/powder ratio of 0.32 and for 30 sec, and then poured into the acrylic ring. After mixing for 14 min, the acrylic ring was placed under a dial gauge as shown in Fig. 2.

The expansion of outer diameter of the acrylic ring was measured, and the results were changed to the expansion of the inner diameter of the ring using Equations 1 and 2 as follows. Let A and B denote the inner and outer radii of the acrylic ring, P_i and P_o the uniform internal and external pressure, E_r and ν Young's modulus and Poisson's ratio, and U the radial displacement. U is given by [4, 5]

$$U = \frac{1-\nu}{E_r}\frac{A^2P_i - B^2P_o}{B^2 - A^2}r + \frac{1+\nu}{E_r}\frac{A^2B^2}{B^2 - A^2}(P_i - P_o)\frac{1}{r} \qquad (1)$$

where r is a distance from the centre of the ring.

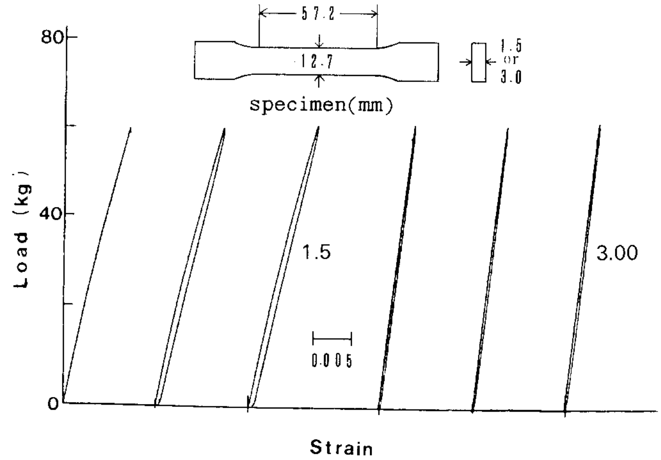

Figure 1 Tensile test of acrylic resin.

Noting that $P_o = 0$, because no external force was applied on the outer wall, the relation between $U(r = A)$ and $U(r = B)$ is given by Equation 2 from Equation 1

$$U(r = A) = U(r = B)\frac{(1 - \nu)A^2 + (1 + \nu)B^2}{2AB} \tag{2}$$

The measurements were carried out up to 120 min from the start of mixing and at 20 ± 2° C.

2.3. Calculation of setting expansion

First, the deformation of the acrylic ring with time, $U_r(r, t)$, is given by Equation 3 by substituting $U_r(r, t)$ for U, 0 for P_o and $P(t)$ for P_i in Equation 1

$$U_r(r, t) = \frac{1}{E_r(B^2 - A^2)} \times \left[(1 - \nu)A^2 r + (1 + \nu)\frac{A^2B^2}{r}\right]P(t) \tag{3}$$

Therefore, $\partial U_r/\partial t$ is given by Equation 4

$$\frac{\partial U_r}{\partial t} = \frac{1}{E_r(B^2 - A^2)} \times \left[(1 - \nu)A^2 r + (1 + \nu)\frac{A^2B^2}{r}\right]\frac{dP(t)}{dt} \tag{4}$$

Second, the setting expansion of investment, $U_i(r, t)$, in the acrylic ring with an inner radius of A and an outer radius of B was obtained as follows. As previously reported [2], a setting expansion of the investment under a stress condition can be obtained by substituting $\partial U_i/\partial t$ for U, $a_0 k \exp(-kt)$ for αT, ν' for ν, and $a_0 k \exp(-kt)/E'$ for $1/E$, where $E' = 5.0\,\mathrm{kg\,cm^{-2}}$, $\nu' = 0.2$, $a_0 = 0.009$, and $k = 0.032\,\mathrm{min^{-1}}$, in the equation that represents the deformation of an elastic body under the same conditions. The deformation of a cylinder of elastic body under radial pressure P, as is the case of the investment in a ring, could be obtained from Equation 1 by substituting $P(t)$ for P_o, 0 for P_i, 0 for A, and A for B. Furthermore, when uniform thermal expansion, αT, is considered, Equation 1 becomes

$$U = -\frac{1 - \nu}{E}rP + r\alpha T \tag{5}$$

Then substitution as described above in Equation 5 results in

$$\frac{\partial U_i(r, t)}{\partial t} = a_0 k \exp(-kt)r \times \left[1 - \frac{1 - \nu'}{E}P(t)\right] \tag{6}$$

Considering that

$$\frac{\partial U_r(A, t)}{\partial t} = \frac{\partial U_i(A, t)}{\partial t}$$

Equation 7 is obtained from Equations 4 and 6

$$L\frac{dP}{dt} = a_0 k \exp(-kt)r \times \left[1 - \frac{1 - \nu'}{E'}P(t)\right] \tag{7}$$

where

$$L = \frac{(1 - \nu)A^2 + (1 + \nu)B^2}{E_r(B^2 - A^2)}$$

The valuables are separated by multiplying by dt and dividing by $\{1 - [(1 - \nu')/E']P\}$ in Equation 7

$$\left[L\Big/\left(1 - \frac{1 - \nu'}{E'}P\right)\right]dp = a_0 k \exp(-kt)\,dt \tag{8}$$

Integrating

$$-L\frac{E'}{1 - \nu'}\ln\left(1 - \frac{1 - \nu'}{E'}P\right) = -a_0 k \exp(-kt) + C \tag{9}$$

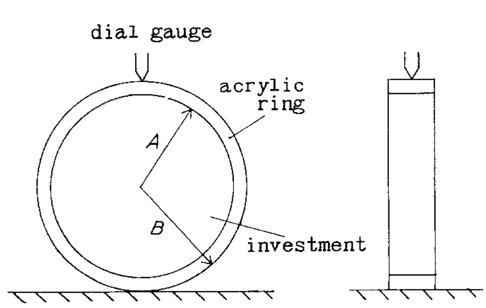

Figure 2 Measurement of setting expansion in an acrylic ring.

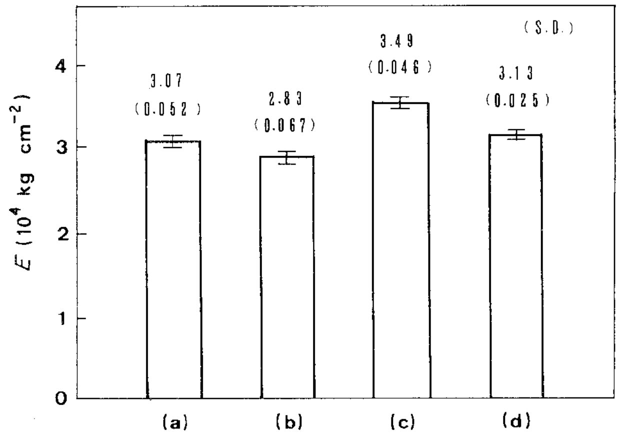

Figure 3 Young's modulus of acrylic resin. (a) 1.5 mm thick, 0.017 mm min^{-1} test speed; (b) 1.5 mm thick, 1.76 mm min^{-1} test speed; (c) 3.0 mm thick, 0.017 mm min^{-1} test speed; (d) 3.0 mm thick, 1.76 mm min^{-1} test speed.

where C is an integration constant. Considering the initial condition that $P(t) = 0$ at $t = 0$, the value of C is determined to be

$$C = a_0 k \tag{10}$$

and the solution of Equation 7 becomes

$$P(t) = \frac{E'}{1 - v'} \times \left(1 - \exp\left\{-\frac{1 - v'}{E'}\frac{a_0}{L}\left[1 - \exp(-kt)\right]\right\}\right) \tag{11}$$

Finally, $U_i(r, t)$ is given by Equation 12 from Equations 6, 7 and 11

$$U_i(r, t) = rLP(t) = \frac{rLE'}{1 - v'}\left(1 - \exp\left\{-\frac{1 - v'}{E'}\frac{a_0}{L}\left[1 - \exp(-kt)\right]\right\}\right) \tag{12}$$

That is, the setting expansion of the investment may be calculated from Equation 12.

3. Results and discussion

Fig. 1 shows tensile load–strain curves of the acrylic resin. The applied load increased once from 0 to 60 kg, and then reduced to 0 after loading. The results indicated that little strain remained when the applied stress was unloaded, thus the acrylic resin can be treated as an elastic body. The values of Young's modulus, E, were obtained from the slopes of these curves, and they are shown in Fig. 3. The value of E was affected by both cross-head speed and specimen size [6]. In this study, $E = 30\,000$ kg cm^{-2} for the acrylic resin was used in all calculations, and the standard deviation of E was about 3000. Fig. 4 shows the change of setting expansion within the acrylic ring. In all cases the value of setting expansion reduced with time compared with unrestricted expansion, because the force which acted on the investment in the ring increased as setting expansion proceeded with time.

Fig. 5 shows the amount of internal pressure, P, which was calculated using Equation 12 at 120 min. The setting expansion in the ring 3 mm thick did not proceed further once the internal pressure reached about 7 kg cm^{-2}, as shown in Fig. 4 which also shows the setting expansion curves in the acrylic ring according to Equation 12. After obtaining the final values of setting expansion, both measured and calculated, the two values were seen to be in good agreement for the 3 mm thick ring but a difference of 6 to 8% was found for ring thicknesses of 0.3 and 1.0 mm. The change in setting expansion with time agreed fairly well for both 0.3 and 1.0 mm thicknesses, but the calculated expansion increased more rapidly than the measured one, especially in the case of the 3.0 mm thick ring. The calculated and measured values agreed fairly well in this way, and our method would be useful for estimating a value of setting expansion under various kinds of restrictive force conditions. For example, the effect of mechanical properties of the wax pattern materials on the value of setting expansion of the investment was estimated here. It was reported that dental waxes have various values of Young's modulus, e.g. 1330 kg cm^{-2} for Kerr Green Casting, 7070 kg cm^{-2} for Kerr Regular Inlay, and 9870 kg cm^{-2} for Peck's Purple, at 23° C [7]. Therefore, three values of E, such as 1000, 5000 and 10 000 kg cm^{-2}, which correspond to those of waxes, were adopted, and the setting expansion in an elastic ring with these values of E and with 10 mm inner diameter and 12 mm outer diameter were calculated using Equation 12. Poisson's ratio of the waxes was

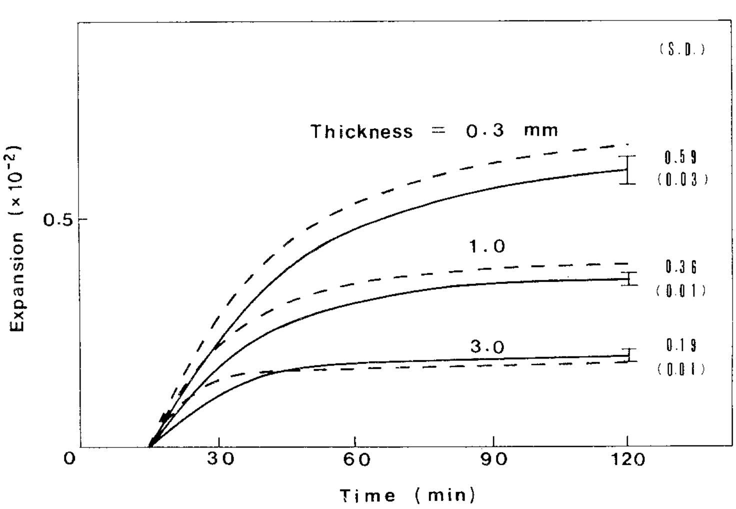

Figure 4 Setting expansion in an acrylic ring: (——) measured, (- - -) calculated.

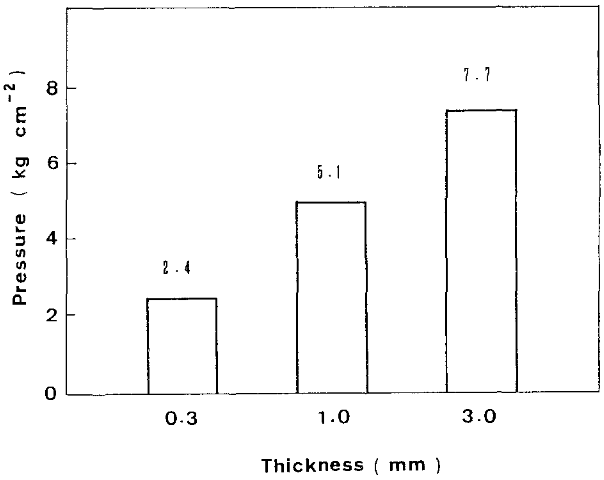

Figure 5 Internal pressure at 120 min calculated from the deformation of the ring.

assumed to be 0.3 in these calculations. The calculated expansion curves are shown in Fig. 6. These results suggest that the type of wax would affect the setting expansion and, finally, the accuracy of dental casting, and it was reported that the setting expansion of the investment in a casting ring varies according to the type of wax used [8, 9]. These reported results cannot be compared with our calculated values directly because our calculations were limited to a setting expansion within an elastic ring in this study. However, it would be useful to apply the method introduced here to setting expansions under more practical conditions in dental casting.

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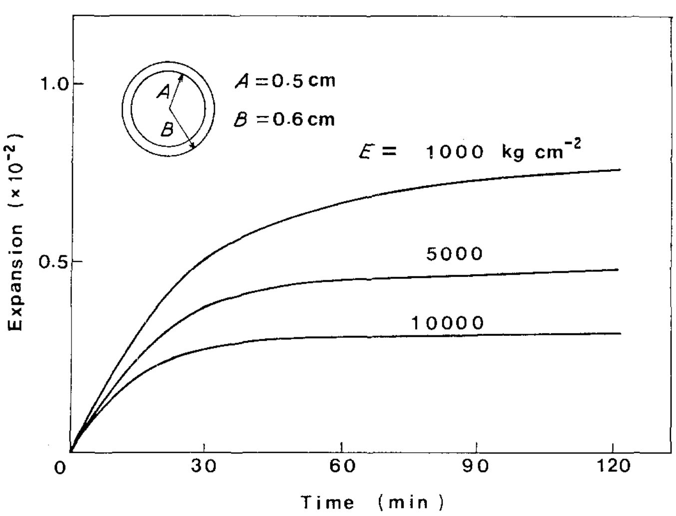

Figure 6 Setting expansion in an elastic ring with various values of E using Equation 12.